## Angular velocity, angular speed, and acceleration

**Example**

A car's tachometer records the engine speed is 3000 revolutions per minute. What is this in $\text{rads}^{-1}$?

**Example**

It takes light 8 minutes and 20 seconds to reach the earth from the sun. How fast is the earth moving?

**Fact —** A particle moving in a circle of radius $r$ with angular speed $\omega$ has tangential speed $v$, given by $v = r\omega$.

**Example**

The pilot of an aircraft flying at $800\,\text{kmh}^{-1}$ on a bearing of $250°$ receives orders to change course to $210°$. The manoeuvre is completed in 20 seconds. Calculate the radius of the turn.

**Fact —** A particle moving in a circle of radius $r$ with constant angular speed $\omega$ and tangential speed $v$

has an acceleration of magnitude $r\omega^2$, or $\frac{v^2}{r}$, towards the centre of the circle.

**Example**

Copse corner has a radius of 440 m. The fastest drivers take the corner at 290 $\text{kmh}^{-1}$. What acceleration do they experience during the corner?

**Example**

A particle of 150 g moves in a horizontal circle of radius 50 cm at a constant speed of 4 $\text{ms}^{-1}$. Find the force towards the centre of the circle that must act on the particle.

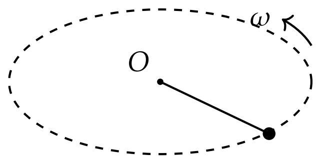
# Circular Motion in two dimensions

**Example**

A particle of mass 300 g is attached to one end of a light in extensible string of length 40 cm, the other endof the string being fixed at $O$ on a smooth horizontal sufrace. If the particle describes circles, centre $O$, find the tension in the string when

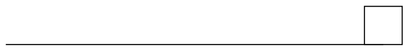
(a) the speed of the particle is $2\sqrt{2}\text{ms}^{-1}$,

(b) the angular speed of the particle is $5\,\text{rads}^{-1}$



**Example**

A car of mass $M$ kg is travelling round a bend which is an arc of a circle radius 140 m. The greatest speed at which the car can travel round the bend without slipping is $45\,\text{kmh}^{-1}$. Find the coefficient of friction between the tyres of the car and the road.

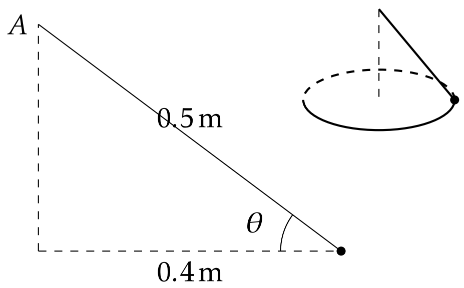


# Horizontal Circular Motion in three dimensions

## Conical Pendulum

**Example**

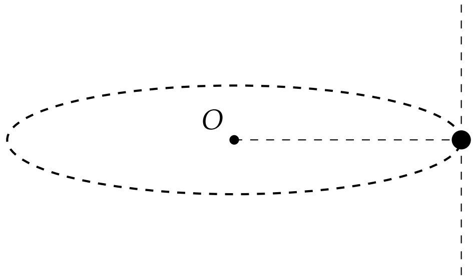
A particle of mass 2 kg is attached to one end of a light inextensible string of length 50 cm. The other end of the string is attached to a fixed point $A$. The particle moves with constant angular speed in a horizontal circle of radius 40 cm. The centre of the circle is vertically below $A$. Calculate the tension in the string and the angular speed of the particle.



## Banked Corner

**Example**

An aircraft of mass 2 tonnes flies at 500 $\mathrm{km\,h^{-1}}$ on a path which follows a horizontal circular arc in order to change course from due north to due east. The aircraft turns in the clockwise direction from due north to due east. It takes 40 s to change course, with the aircraft banked at an angle $\alpha$ to the horizontal. Calculate the value $\alpha$ and the magnitude of the lift force perpendicular to the surface of the aircraft's wings.

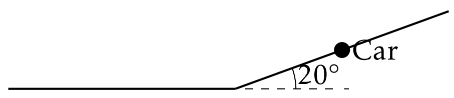


**Example**

A racing track has a circular bend of radius 150 m banked at an angle of 20° to the horizontal. A car of mass 800 kg travels around this bend. Find:

(a) the speed at which the car experiences no sideways friction

(b) the normal reaction on the car at this speed



**Example** (OCR M2 January 2010 Q7)

A particle $P$ of mass 0.2 kg is moving on the smooth inner surface of a fixed hollow hemisphere which has centre $O$ and radius 5 m. $P$ moves with constant angular speed $\omega$ in a horizontal circle at a vertical distance of 3 m below the level of $O$. (See Fig. 1)

(i) Calculate the magnitude of the force exerted by the hemisphere on $P$ [3]

(ii) Calculate $\omega$ [4]

A light inextensible string is how attached to $P$. The string passes through a small hole at the end of the lowest point of the hemisphere and a particle of mass 0.1 kg hangs in equilibrium at the end of the string. $P$ moves in the same horizontal circle as before (see Fig. 2)

(iii) Calculate the new angular speed of $P$ [8]